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A Theory of the Effect of Corporate Income Tax on the Optimal Output of a Firm Facing an Uncertain Demand

Edmund H. Mantell
Professor of Finance and Economics
Lubin School of Business, Pace University

ABSTRACT

Most professional and public discussions of the reduction of the corporate income tax rate in the United States (enacted in year 2017) focus on its income-distributive effects. This article addresses the question how, if at all, the change in the tax rate can be expected to affect a firm's decision with respect to its optimal output and consequent deployment of factor inputs. The theory developed in this article assumes the firm faces an uncertain demand function embodying an additive random variable. The article derives three propositions relating the change in the tax rate and the firm's output to the firm's attitude towards risk.

Key words: corporate income tax, risk , corporate output

INTRODUCTION

From 1993 until very recently the maximum marginal corporate income tax rate in the United States has been 35 percent. In some earlier years it has been much larger.[1] On December 20, 2017, the US Senate and House of Representatives passed the *Tax Cut and Jobs Act* (i.e. the TCJA).[2] Among other provisions of the *Act*, it set an effective corporate tax rate of 21 percent on all corporate net income, effective on January 1, 2018.

Before and after the enactment of the TCJA there have been discussions in the professional and academic literature, as well as in the popular press, addressing the economic effects of the corporate income tax reduction. Most of the discussions have focused on income-distributive effects of the tax reduction: *viz* increases in cash dividends paid to stockholders, increases in stock repurchases and increases in bonuses and/or wages paid to employees.[3] However, an analysis of the economic effects of a reduction in the corporate income tax rate should reflect not only on how companies decide to allocate the increase in their after-tax income. It should reflect how corporations change their behavior in response to what they believe to be a permanent tax cut.

Absent from the professional literature is an analysis of how, if at all, the change in the corporate tax rate can be expected to affect the profit-maximizing output of a firm facing an

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[1] In 1952 the top corporate marginal tax rate was 52 percent and in 1968 it was 52.8 percent.

[2] Public law no. 115-97, an *Act to provide for reconciliation pursuant to titles II and V of the concurrent resolution on the budget for fiscal year 2018*, is a congressional revenue act originally introduced in Congress as the *Tax Cuts and Jobs Act*.

[3] A news report appearing in the *New York Times* on February 28, 2018 (p. B1) exemplifies the issues addressed in the popular press: "*President Trump promised that his tax cuts would encourage companies to invest in factories, workers and wages, setting off a spending spree that would reinvigorate the American economy. Companies have announced plans for some of those investments. But so far, companies are using much of the money for something with a more narrow benefit: buying their own shares*."

uncertain demand. The motivation for this analysis is a remark in an article by Applebaum and Katz [2,p. 528.]:

> *"… it may be interest to note the effects of changes in a proportional profits tax (with full loss offset)…. While space consideration [in their publication] preclude the presentation of the long-run analysis, it appears that it is not possible to sign the long-run effect of a proportional profits tax on any of the variables under consideration."*

The analysis carried out in this paper will address the question raised in the paragraph above. This paper applies an analytical model supporting inferences as to the algebraic sign of the effect of a change in a profits tax rate on the output of a firm facing an uncertain demand curve.

Section II develops the formal model of a firm facing a downward sloping demand curve embodying an additive random variable. I assume the firm sets its output in such a way as to maximize the expected value of the risk-adjusted profits function. That risk-adjustment is assumed to be a utility function with the usual properties.

Apparently the firm's optimizing behavior in this kind of theoretical scenario is not settled within the economics profession. Two recent publications exemplify the differences of professional opinion

In the paper by Driver and Valletti [5, p. 187] the authors analyzed the differences in the optimal pricing implications for a monopoly firm facing an uncertain demand, where a random factor can enter either additively or multiplicatively. Those authors concluded:

> *"…. for additive uncertainty [in the demand function] the [optimal] price should be lower than the certainty level [price.]"*

In the recently published paper by Gajek, et. al. [6, p. 287]. those authors reach a similar conclusion:

> *"Our model indicates that a monopolist facing the risk of demand uncertainty will respond with lower price and sacrifice some level of profits available in the absence of uncertainty."*

A main result of this paper is manifestly inconsistent with the conclusions reproduced above. Proposition 1 in this paper shows that if the uncertainty enters the demand function as an additive random variable, and the firm is risk averse, the firm's optimal price will be higher than the certainty price because the firm's optimal output will be smaller than the certainty output.

In Section II the firm's' profits are not taxed. The model establishes propositions explaining how the demand uncertainty affects the firm's optimal output in the absence of a corporate profits tax. It is shown that the optimal outputs reflect the firm's alternative attitudes towards risk.

Section III extends the model by introducing a corporate profits tax. The analysis leads to three propositions relating sign of the tax-induced changes in the firm's output to its attitude towards risk irrespective of the tax rate.

Section IV consists of concluding remarks. The tax policy implications of the theoretical results are briefly discussed.

THE MODEL OF FIRM BEHAVIOR IN THE ABSENCE OF INCOME TAX WHEN IT FACES AN UNCERTAIN DEMAND

Consider a firm operating in a market where it enjoys significant market power. It faces a downward sloping demand function of the form:

$$P = D(Q, x) \tag{1}$$

where Q is the quantity of the firm's output offered for sale, P is the unit price, $\frac{\partial}{\partial Q} D(Q, x) < 0$ and x is a randomly distributed variable.

It is assumed that the random variable x is governed by the cumulative probability distribution: $F(x^*) = Prob(x \leq x^*)$. The expected value of x is symbolized by $\bar{x}$. The variance of x is symbolized by σ^2.

The function $C(Q)$ represents the firm's total cost function at output level Q.

The firm's single period profits are calculated as:

$$\pi(Q, x) = D(Q, x)Q - C(Q) \tag{2}$$

The firm's attitude towards risk is embodied in the utility function of the firm's profits, symbolized by $U[\pi(Q)]$. The firm's attitude towards risk is indicated by a change in the marginal utility when profit varies.[4] The paper by Baron [3, p. 203] and the book by Milgrom and Roberts [10, p. 247] use a Taylor's series expansion to represent the function $U[\pi(Q)]$ with respect to the point $\bar{x}$.

$$U(\pi) = \sum_{i=0}^{\infty} a_i \, [\pi(Q, x) - \pi(Q, \bar{x})]^i \tag{3}$$

The truncated Taylor series in (3) can be expressed as:

$$U(\pi) = \pi(Q, x) + \frac{\theta}{2}[\pi(Q, x) - \pi(Q, \bar{x})]^2 + R(Q, x) \tag{4}$$

The function $R(Q, x)$ symbolizes the sum of the remaining terms in series (3) for all values of $i \geq 2$. Milgrom and Roberts explicitly assume that remainder is negligible. Adopting the same assumption, we can calculate the utility of the firm's profits function as:

$$U(\pi) \cong \pi(Q, x) + \frac{\theta}{2} [\pi(Q, x) - \pi(Q, \bar{x})]^2 \tag{5}$$

The parameter θ is assumed to be constant.

The expected value of the firm's utility function is calculated as:

$$E[U(\pi)] = \int \pi(Q, x) dF(x) + \frac{\theta}{2} \int [\pi(Q, x) - \pi(Q, \bar{x})]^2 dF(x) \tag{6}$$

[4] This is the definition of a firm's attitude towards risk appearing in the paper by Hawawini [8 , p. 195] the same definition is found in Bauer [4 , p. 6]

The expectation in (6) can be resolved into a specific functional form only if a simplifying assumption supports an analytical expression for the way in which the random variable x enters the demand function. It is assumed the random variable enters additively: [5]

$$P = D(Q) + x \tag{7}$$

I adopt two additional assumptions describing the properties of x:

(i) It is assumed that the domain of x is bounded from below: $F\big(-D(0)\big) = 0$

(ii) It is assumed $\bar{x} = 0$.

Mathematical Appendix A proves that the additive property of x and assumptions (i) and (ii) imply the expected value of the firm's utility function can be calculated as:

$$E[U(\pi)] = \pi(Q) + \frac{\theta}{2} Q^2 \sigma^2 \tag{8}$$

where $\pi(Q) = D(Q)Q - C(Q)$. The function $\pi(Q)$ is the firm's expected profits function. It is formally equivalent to a certain demand curve. Leland describes it as "*…the demand curve which would result if the firm knew price would equal its expected value with certainty for all levels of Q.*" [6] The risk contemplated by the firm is represented by the variance σ^2.

The parameter θ is derived from the properties of the derivatives of the firm's utility function. It is a manifestation of the firm's attitude towards risk and is assumed to be invariant with respect to the firm's profits. Its sign reflects the following attitudes:

$$\theta \begin{cases} < 0 & \text{implies the firm is risk averse} \\ = 0 & \text{implies the firm is indifferent to risk} \\ > 0 & \text{implies the firm is risk} - \text{seeking} \end{cases}$$

The firm's objective function is $E[U(\pi)]$. The firm's only control variable is its output. Thus, the firm will choose a level of output that maximizes $E[U(\pi)]$.[7]

The first-order condition is found by taking the derivative of $E[U(\pi)]$ with respect to Q, setting it equal to zero and solving the resulting equation.

$$\frac{d}{dQ} E[U(\pi)] = \frac{d}{dQ}[\pi(Q)] + \theta Q \sigma^2 = 0 \tag{9}$$

If the first order condition is satisfied, equation (9) can be expressed as:

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5 The plausibility of an additive random variable in a demand function has been compared with an alternative specification consisting of a multiplicative random variable. In the latter model, the certainty demand curve is multiplied by a randomly distributed shift term. See, for example, the article by Driver [5, p. 188]. The article by Aiginger [1 , p. 166] considered the additive and the multiplicative models with a view to inferring which model has a better claim to realism. He concluded that there was no rational economic basis to choose between the specifications.

6 Leland [9 , p. 281]. Equation (8) is very similar to the certainty equivalent expression derived in Milgrom and Roberts [10 , p. 247]

7 The assumption that the a firm chooses its output in such a way as to maximize the expected utility of its profits is identical to the firm's behavior assumed in the paper by Hau [7, p. 458]

$$MR = MC - \theta Q\sigma^2 \qquad (10)$$

where MR represents the marginal revenue to the firm at its optimal output and MC represents the firm's marginal cost at the same output.

Equation (10) can be used to infer the profit-maximizing price and output configuration of a firm manifesting different attitudes towards risk. The inferences are summarized below.

(a) The firm is indifferent to risk.

This case corresponds to a value of $\theta = 0$. In this case the firm sets its output to be same it would choose if the expected value of the demand function was identical to the certainty demand function.

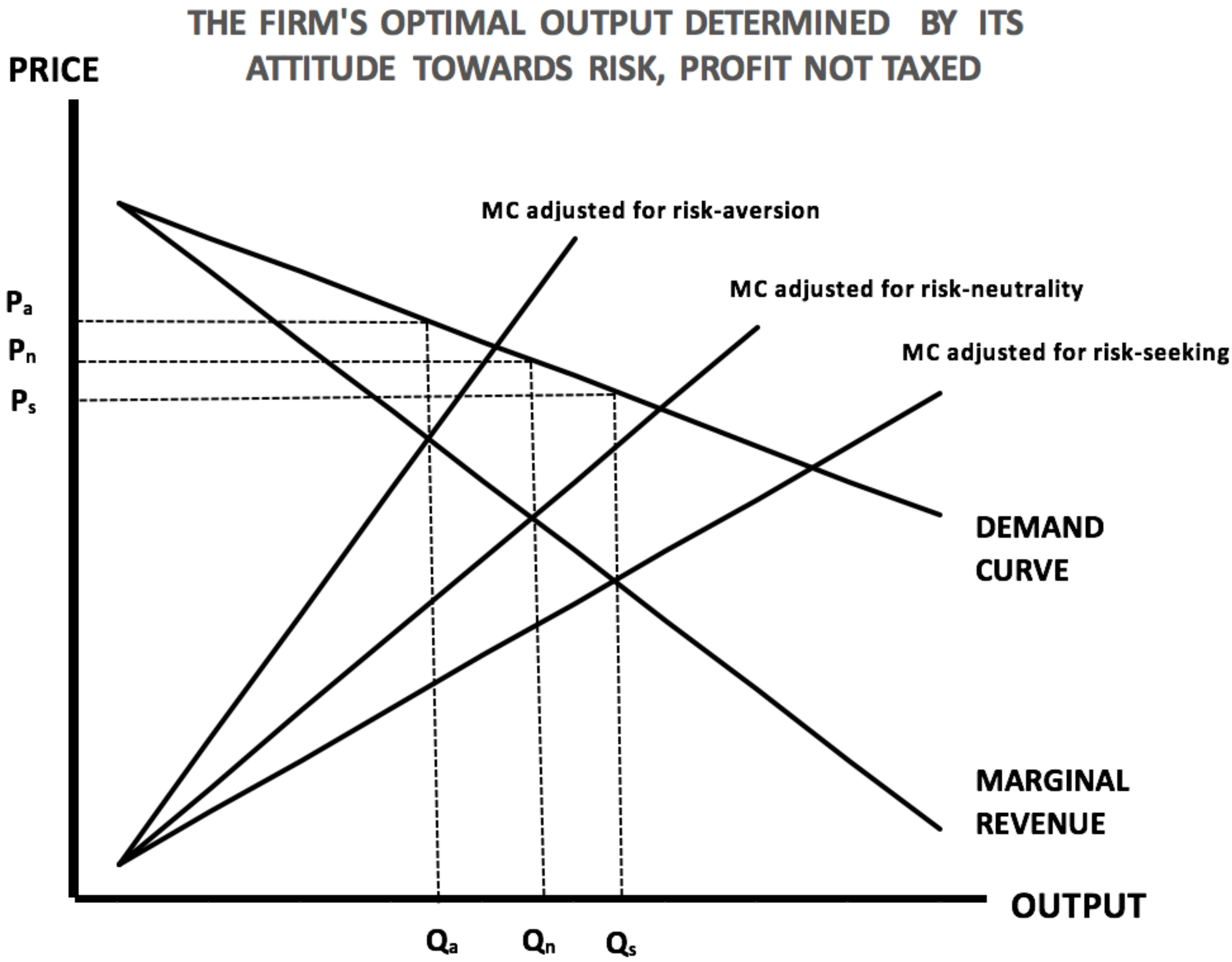
(b) The firm is risk-averse.

This case corresponds to a value of $\theta < 0$. In this case the firm sets its output where $MR > MC$ for any $Q > 0$. The effect of the firm's risk aversion is to cause it to set a smaller output and a higher price than it would have set in the absence of uncertainty.

(c) The firm is risk-seeking.

This case corresponds to a value of $\theta > 0$. In this case the firm sets its output where $MR < MC$ for any $Q > 0$. The effect of the firm's risk seeking is to cause it to set a larger output and a lower price than it would have in the absence of uncertainty.

The diagram below illustrates the effect on the firm's optimal output and price of differences in its attitude towards risk (i.e. variations in θ) when the firm is conducting business in a jurisdiction where its profits are not taxed.



The diagram relates the optimal output with the firm's price at each of the three attitudes towards risk. The symbol Q_a represents the optimal output for a risk-averting firm; The symbol Q_n represents the optimal output for a risk-neutral firm; The symbol Q_s represents the optimal output for a risk-seeking firm. The prices P_a, P_n and P_s are associated with each level of output. We see: $Q_a < Q_n < Q_s$ and $P_a > P_n > P_s$.

We can examine the effect on the optimal output of small changes in the value of θ. Let the symbol Q^* represents the optimal output for any value of θ. Mathematical Appendix B derives the algebraic sign of $\frac{dQ^*}{d\theta}$ and shows that it is positive. However, the firm's output decision in response to a change in its attitude towards risk is determined by the algebraic sign of θ. The result can be summarized in the following Proposition 1:

Proposition 1: For a firm facing a downward sloping demand curve embodying an additive random variable, when the firm is operating in a jurisdiction where its profits are not taxed:

(a) *If the firm is risk-averse, its optimal output approaches the risk-neutral output as the firm's aversion to risk decreases (i.e. the negative value of* θ becomes less negative.)

(b) *If the firm is risk-neutral, its optimal output is the same as it would be for a certainty demand curve.*

(c) *If the firm is risk-seeking its optimal output becomes increasingly larger than a risk-neutral output as its appetite for risk increase (i.e. the positive value of* θ becomes larger.)

THE EFFECT OF A CORPORATE INCOME TAX ON THE OPTIMAL OUTPUT OF THE FIRM FACING AN UNCERTAIN DEMAND

Economists recognize that a reduction in the rate at which corporate income is taxed can have two different consequences on corporate behavior.

An increase in the corporation's after-tax income can be manifested in the distributive effects described in the Introduction. Arguably much more significant in the long run are the effects of the tax rate reduction on firms' investment policy. A tax cut increases the incentive of a firm to invest. A lower corporate tax rate gives investors in new plant and equipment a larger share of the net income those investments generate. That larger share leads more potential investments to satisfy the capital budgeting criteria that tell a company whether potential investments are like to enrich the stockholders in the long run.[8]

A third effect of a reduction in the corporate tax rate does not seem to have received attention from analytical models. That is the effect on the firm's output decision in the face of uncertainty if the tax rate is changed. The model developed above will be applied to analyze how a tax imposed on corporate profits can be expected to affect firm's output.

Suppose the average effective tax rate applicable to the firm's profits is symbolized by T and the firm's after-tax profits are symbolized by $\pi_T(Q,x)$. The after-tax profits are calculated as:

$$\pi_T(Q,x) = (1-T)[D(Q,x)Q - C(Q)] \tag{11}$$

If one assumes the tax rate is fixed, the Taylor series expansion results in the approximation given by equation (12). It is the after-tax counterpart to equation (8).

$$E[U(\pi_T)] \cong (1-T)\pi(Q) + (1-T)^2\frac{\theta}{2}Q^2\sigma^2 \tag{12}$$

Taking the derivative of equation (12) with respect to Q, setting it equal to zero and solving, we have the first-order condition for an after-tax maximum in equation (13).

$$MR = MC - (1-T)\theta Q\sigma^2 \tag{13}$$

Comparing equation (13) with equation (10) we can infer the effect of a corporate profits tax on the profit-maximizing output of a firm facing an uncertain demand curve. The after-tax coefficient $(1-T)$ is less than one and thus it narrows the gap between the firm's profit-maximizing marginal revenue and its marginal cost, regardless of the magnitude of θ. Thus, the imposition of a fixed tax rate on corporate profits will tend to mitigate the variation in output caused by the firm's alternative attitudes towards risk. This inference can be summarized in Proposition 2.

Proposition 2: For a firm facing a downward sloping demand curve embodying an additive random variable, if the firm's profit is taxed at a fixed rate, the effect of the tax is to lessen the deviation of the firm's output from the output of a risk neutral firm, irrespective of the firm's attitude towards risk.

8 The reasoning in this paragraph supports the President's assertion that the reduction in the corporate income rate *"…is jet fuel for the economy."*

The implication of Proposition 2 is that if the firm's average effective tax rate is large, the effect of that tax on the firm's optimal output will be to move it closer to the optimal output of a risk-neutral firm.

We can extend Proposition 2 to consider the effect on the firm's output of a change in the parametric tax rate. Mathematical Appendix 3 calculates the total differential of equation (13) to derive the algebraic sign of dQ/dT. The resulting inferences are summarized in Proposition 3.

Proposition 3: For a profit-maximizing firm facing a downward sloping demand curve embodying an additive random variable, if the tax rate applied to the firm's profit decreases, the consequent effect on the firm's optimal output will be determined by the firm's attitude towards risk:

(a) *If the firm is risk averse, the decrease in the tax rate will cause the optimal output to decrease;*
(b) *If the firm is risk-neutral, its optimal output will not be affected by a change in the tax rate;*
(c) *If the firm is a risk seeker, a decrease in the tax rate will cause the optimal output to increase.*

CONCLUDING REMARKS AND TAX POLICY

Propositions 2 and 3 in this paper have tax-policy implications. Both propositions suggest that a decrease in the corporate tax rate (such as was enacted in the TCJA) can be expected to induce firms to increase their output (and hence their deployment of factors of production) only if those firms are risk seekers (i.e. $\theta > 0$).

If firms are risk averse, a reduction in the corporate tax rate can be expected to have the opposite of the effect claimed for it by those who suggest that it will induce firms to invest in expanding their capacity.

If firms are indifferent to risk, a reduction in the corporate tax rate can be expected to have no the effect on the firm's investment to expand their capacity.

To the extent that the public announcements are broadly representative of firms' responses to the reduction in the corporate tax rate, it would appear that the main effect of the TCJA is to transfer corporate wealth to stockholders.

MATHEMATICAL APPENDIX A

The expectation of the truncated Taylor's series is calculated as:

$$E[U(\pi)] = \int \pi(Q,x)dF(x) + \frac{1}{2}\theta \int [\pi(Q,x) - \pi(Q,\bar{x})]^2 dF(x) \qquad A1$$

The first integral can be partitioned and calculated as:

$$\begin{aligned}\int \pi(Q,x)dF(x) &= \int [D(Q)+x]\, QdF(x) - \int C(Q)dF(x) \qquad A2\\ &= \int D(Q)QdF(X) + Q\int xdF(x) - C(Q)\\ &= D(Q)Q + \bar{x}Q - C(Q)\end{aligned}$$

Inasmuch as we assumed $\bar{x} = 0$, the first integral in $A1$ can be written as:

$$\int \pi(Q,x)dF(x) = D(Q)Q - C(Q) = \pi(Q) \qquad A3$$

The second integral in $A1$ can be partitioned and calculated as:

$$\begin{aligned}\int [\pi(Q,x) - \pi(Q,\bar{x})]^2 dF(x) &= \int [D(Q,x)Q - C(Q) - D(Q,\bar{x})Q + C(Q)]^2 dF(x) \qquad A4\\ &= \int [D(Q,x)Q - D(Q,\bar{x})Q]^2\, dF(x)\\ &= \int \{Q[D(Q,x) - D(Q,\bar{x})]\}^2 dF(x)\\ &= Q^2 \int [D(Q,x) - D(Q,\bar{x})]^2 dF(x)\\ &= Q^2 \int [D(Q) + x - D(Q) - \bar{x}]^2\, dF(x)\\ &= Q^2 \int [x - \bar{x}]^2\, dF(x) = Q^2\sigma^2\end{aligned}$$

Thus the second integral in $A1$ can be written:

$$\frac{1}{2}\theta \int [\pi(Q,x) - \pi(Q,\bar{x})]^2 dF(x) = \frac{1}{2}\theta Q^2\sigma^2 \qquad A5$$

Adding the first integral and the second integral derives the result in the text.

MATHEMATICAL APPENDIX B

Taking the total differential of equation (9) with respect to θ, the differential of the left-hand side is calculated as:

$$dMR = \frac{d(MR)}{dQ}\frac{dQ}{d\theta} \qquad B1$$

The total differential of the right-hand side of (9) with respect to θ is calculated as:

$$d[MC - \theta Q\sigma^2] = \frac{d(MC)}{dQ}\frac{dQ}{d\theta} - Q\sigma^2 - \theta\sigma^2\frac{dQ}{d\theta} \qquad B2$$

Setting $B1$ and $B2$ equal we have:

$$\frac{d(MR)}{dQ}\frac{dQ}{d\theta} = \frac{d(MC)}{dQ}\frac{dQ}{d\theta} - Q\sigma^2 - \theta\sigma^2\frac{dQ}{d\theta} \qquad B3$$

Equation $B3$ can be factored and expressed as:

$$\frac{dQ}{d\theta}\left[\frac{d(MR)}{dQ} - \frac{d(MC)}{dQ} + \theta\sigma^2\right] + Q\sigma^2 = 0 \qquad B4$$

The second-order condition for maximization of $E[U(\pi)]$ is calculated as:

$$\begin{aligned}\frac{d^2}{dQ^2}E[U(\pi)] &= \frac{d^2}{dQ^2}[\pi(Q)] + \theta\sigma^2 \\ &= \frac{d(MR)}{dQ} - \frac{d(MC)}{dQ} + \theta\sigma^2\end{aligned} \qquad B5$$

Equation $B5$ is seen to be the bracketed expression in equation $B4$. If maximization of $E[U(\pi)]$ is achieved, the second order condition must satisfy the inequality:

$$\frac{d^2}{dQ^2}E[U(\pi)] < 0 \qquad B6$$

Thus, the bracketed expression in $B4$ is negative. Inasmuch as $Q\sigma^2$ must be positive, equation $B4$ can be satisfied if and only if $\frac{dQ}{d\theta} > 0$. The inequality proves Proposition 1.

MATHEMATICAL APPENDIX C

Taking the total differential of equation (13) with respect to T, the differential of the left-hand side is calculated as:

$$dMR = \frac{d(MR)}{dQ}\frac{dQ}{dT} \qquad C1$$

The differential of the right side of equation (13) is

$$d[MC - (1-T)\theta Q\sigma^2] = \frac{d(MC)}{dQ}\frac{dQ}{d\mathrm{T}} - \theta\sigma^2\frac{dQ}{dT} + \theta Q\sigma^2 + T\theta\sigma^2\frac{dQ}{dT} \qquad C2$$

Equation $C2$ can be factored:

$$d[MC - (1-T)\theta Q\sigma^2] = \frac{dQ}{dT}\left[\frac{d(MC)}{dQ} - \theta\sigma^2(1-T)\right] + \theta Q\sigma^2 \qquad C2$$

Setting $C1$ equal to $C2$, we have:

$$\frac{d(MR)}{dQ}\frac{dQ}{dT} = \frac{dQ}{dT}\left[\frac{d(MC)}{dQ} - \theta\sigma^2(1-T)\right] + \theta Q\sigma^2 \qquad C3$$

Rearranging terms in equation $C3$, we have:

$$\frac{dQ}{dT}\left[\frac{d(MR)}{dQ} - \frac{d(MC)}{dQ} + \theta\sigma^2(1-T)\right] - \theta Q\sigma^2 = 0 \qquad C4$$

The expression in the brackets in equation $C4$ is the second-order condition that must be satisfied for a value of Q corresponding to the maximum value of $E[U(\pi_T)]$. Thus, if $E[U(\pi_T)]$ is maximized with respect to Q, we may infer the inequality:

$$\frac{d(MR)}{dQ} - \frac{d(MC)}{dQ} + \theta\sigma^2(1-T) < 0 \qquad C5$$

The product term $Q\sigma^2$ in the inequality $C4$ must be positive. Thus, if equation $C4$ is satisfied, the algebraic sign of $\frac{dQ}{dT}$ is determined by the sign of θ. Specifically, the sign of $\frac{dQ}{dT}$ is the opposite sign of θ:

$$\frac{dQ}{dT} \begin{cases} > 0 & if\ \theta < 0 \\ = 0 & if\ \theta = 0 \\ < 0 & if\ \theta > 0 \end{cases}$$

The economic significance of the inequalities is expressed in Proposition 3.

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